A Rank-3 Counterexample in Dimension 10 to the Pentagon Measurement-Order Contextuality Conjecture

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November 27, 2026

Abstract

Wu (arXiv:2611.24604v1, Conjecture 2) recently conjectured that for any five orthogonal projectors $P_1, \dots, P_5$ of arbitrary rank on $\mathbb{C}^d$ satisfying cyclic neighbor orthogonality $P_i P_{i+1} = 0$ ($i \in \mathbb{Z}_5$), the Klyachko–Can–Binicioğlu–Shumovsky (KCBS) contextuality excess $\lambda_{\max}(\sum_{i=1}^5 P_i) - 2$ is bounded above by the largest product of total-variation measurement-order scores across crossing chords, $\max_{i\in\mathbb{Z}_5} \mathcal{T}(P_i, P_{i+2})\,\mathcal{T}(P_{i+1}, P_{i+3})$, where $\mathcal{T}(P,Q) = \|[P,Q]\|_\infty$. While Wu proved this bound for rank-1 projectors on $\mathbb{C}^3$ and direct sums thereof, we construct an explicit counterexample in the $(d, \mathrm{rank}) = (10, 3)$ regime consisting of five rank-3 real symmetric orthogonal projectors on $\mathbb{R}^{10} \subset \mathbb{C}^{10}$ generated by compact integer spanning matrices. Using exact rational arithmetic, an integer Rayleigh witness vector, and exact pivoted $LDL^\top$ positive-semidefinite certificates $t_i^2 I_{10} + [P_i, P_{i+2}]^2 \succeq 0$, we establish a rigorous rational additive violation $\lambda_{\max}(\sum_{i=1}^5 P_i) - 2 - \max_{i\in\mathbb{Z}_5} \mathcal{T}(P_i, P_{i+2})\,\mathcal{T}(P_{i+1}, P_{i+3}) \geq V > 3.031 \times 10^{-3}$, corresponding to a certified ratio lower bound of 1.02214 relative to the rational upper bounds t_i (and a numerical true operator-norm ratio of ≈ 1.02303). This disproves the arbitrary-rank conjecture for $d \geq 10$, while leaving the rank-1 case for $d \geq 4$, lower dimensions $4 \leq d \leq 9$, and Wu's qutrit rank-1 theorem unaffected.

1 Introduction and Problem Statement

In the graph-theoretic framework for Kochen–Specker contextuality [1, 2], five binary projective compatibility events arranged along the 5-cycle C_5 are represented on a finite-dimensional Hilbert space $\mathbb{C}^d$ by orthogonal projectors $P_1, P_2, P_3, P_4, P_5 \in M_d(\mathbb{C})$ satisfying $P_i = P_i^\dagger = P_i^2$ and cyclic orthogonality

$$P_i P_{i+1} = 0 \qquad \text{for all } i \in \mathbb{Z}_5, \tag{1}$$

where indices are taken modulo 5 ($P_6 \equiv P_1$). Every noncontextual hidden-variable model satisfies $\sum_{i=1}^5 p_i \leq \alpha(C_5) = 2$, whereas quantum mechanics achieves up to Lovász's theta number $\vartheta(C_5) = \sqrt{5}$ [1–3]. Consequently, the spectral excess

$$\lambda_{\max}\left(\sum_{i=1}^5 P_i\right) - 2$$

quantifies the maximal state-dependent violation of the KCBS pentagon noncontextuality inequality.

Recently, Wu [4] investigated quantitative upper bounds on uncertainty, contextuality, steering, and Bell nonlocality in terms of sequential measurement-order dependence. For two sharp binary observables $\{P, I_d - P\}$ and $\{Q, I_d - Q\}$ implemented via sequential Lüders instruments on $\mathbb{C}^d$, the total-variation measurement-order score is defined by

$$\mathcal{T}(P,Q) := \max_{\rho \in \mathcal{D}(\mathbb{C}^d)} \mathrm{TV}\big(p^\rho_{PQ}, p^\rho_{QP}\big), \qquad \mathrm{TV}(p,q) := \frac{1}{2}\sum_x |p_x - q_x|.$$

Writing $A = I_d - 2P$ and $B = I_d - 2Q$ for the associated Hermitian involutions, Theorem 1 of Ref. [4] establishes via Jordan's two-projections lemma that in every finite dimension d,

$$\mathcal{T}(P,Q) = \frac{1}{4}\|[A,B]\|_\infty = \|[P,Q]\|_\infty, \tag{2}$$

where $\|\cdot\|_\infty$ denotes the operator (spectral) norm.

For rank-1 projectors on $\mathbb{C}^3$ and direct sums thereof, Wu proved (Ref. [4], Theorem 10) that the KCBS violation is bounded by the largest product of order scores along two crossing chords of the pentagon, with equality $\sqrt{5}-2 = (\sqrt{\sqrt{5}-2})^2 \approx 0.236068$ at the symmetric qutrit KCBS pentagon, and conjectured the same inequality for projectors of arbitrary rank in every dimension:

Conjecture (Wu [4], Conjecture 2). *For all finite dimensions d and all orthogonal projectors $P_1, \ldots, P_5 \in M_d(\mathbb{C})$ of arbitrary rank satisfying $P_i = P_i^\dagger = P_i^2$ and $P_iP_{i+1} = 0$ ($i \in \mathbb{Z}_5$),*

$$\lambda_{\max}\left(\sum_{i=1}^{5} P_i\right) - 2 \le \max_{i\in\mathbb{Z}_5} \mathcal{T}(P_i, P_{i+2})\,\mathcal{T}(P_{i+1}, P_{i+3}). \tag{3}$$

Local numerical searches reported in Ref. [4] around embedded KCBS configurations (including rank-3 projectors in $d = 9$) yielded left-to-right ratios at most 0.9998, leaving Wu's Conjecture 2 open beyond rank-1 qutrit direct sums.

In this note, we show that Wu's Conjecture 2 fails in its unrestricted arbitrary-rank form when $(d, \text{rank}) = (10, 3)$.

Theorem 1 (Disproof of Wu's Conjecture 2 at $(d, \text{rank}) = (10, 3)$)**.** *There exist five real symmetric orthogonal projectors $P_1, P_2, P_3, P_4, P_5 \in M_{10}(\mathbb{Q}) \subset M_{10}(\mathbb{C})$, each of rank 3, satisfying $P_i^\top = P_i = P_i^2$ and cyclic orthogonality $P_iP_{i+1} = 0$ for all $i \in \mathbb{Z}_5$, such that*

$$\begin{aligned}\lambda_{\max}\left(\sum_{i=1}^{5} P_i\right) - 2 &\ge \frac{12190448088494396667226710395562575079 07}{87127486824200215316298854938646553496 35} \\ &\approx 0.1399150662,\end{aligned} \tag{4}$$

$$\max_{i\in\mathbb{Z}_5} \mathcal{T}(P_i, P_{i+2})\,\mathcal{T}(P_{i+1}, P_{i+3}) \le \frac{34221}{250000} = 0.136884, \tag{5}$$

and hence Wu's Conjecture 2 (inequality (3)) is violated by at least the exact rational margin

$$\begin{aligned}V &:= \frac{13204459102528719693522896807044012513781 33}{43563743412100107658149427469323276748175 0000} \\ &\approx +0.0030310662 > 0.\end{aligned} \tag{6}$$

By zero-padding $P_i \mapsto P_i \oplus 0_{d-10}$, the same construction immediately yields rank-3 counterexamples on $\mathbb{C}^d$ for every integer $d \ge 10$.

2 Explicit Integer-Lattice Construction

Because $\mathbb{R}^{10} \subset \mathbb{C}^{10}$ and a real symmetric matrix satisfies $P_i^\dagger = P_i^\top = P_i$, a purely real rational counterexample rigorously falsifies the complex statement of Wu's Conjecture 2. We specify each rank-3 projector $P_i \in M_{10}(\mathbb{Q})$ via a full-column-rank integer spanning matrix $B_i \in \mathbb{Z}^{10\times 3}$ through the standard orthogonal projection formula

$$P_i = B_i\big(B_i^\top B_i\big)^{-1} B_i^\top \qquad (i = 1, \ldots, 5). \tag{7}$$

In the standard basis $\{e_1, \ldots, e_{10}\}$ of $\mathbb{R}^{10}$, fix P_1 and P_2 on disjoint coordinate triples,

$$B_1^\top = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad B_2^\top = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (8)$$

so $B_1^\top B_2 = 0$. Next, define $B_3, B_5 \in \mathbb{Z}^{10\times 3}$ of maximum entry height 19 supported outside $\{4,5,6\}$ and $\{1,2,3\}$, respectively:

$$\begin{aligned} B_3^\top &= \begin{pmatrix} 7 & 12 & 14 & 0 & 0 & 0 & -10 & -4 & 11 & -2 \\ 13 & 3 & 4 & 0 & 0 & 0 & 9 & 4 & -2 & 17 \\ 3 & 8 & 2 & 0 & 0 & 0 & -13 & 5 & -19 & 1 \end{pmatrix}, \\ B_5^\top &= \begin{pmatrix} 0 & 0 & 0 & 3 & -14 & 2 & 10 & -13 & -12 & -1 \\ 0 & 0 & 0 & 13 & -9 & 1 & 0 & 9 & -1 & 17 \\ 0 & 0 & 0 & 3 & -13 & 7 & -3 & -6 & 19 & -4 \end{pmatrix}. \end{aligned} \quad (9)$$

By support disjointness, $B_2^\top B_3 = 0$ and $B_5^\top B_1 = 0$ hold identically. Finally, the joint orthogonal complement $\ker(B_3^\top) \cap \ker(B_5^\top) \subset \mathbb{Q}^{10}$ has dimension 4 and is parametrized freely by coordinates (x_7, x_8, x_9, x_{10}). Choosing on those four coordinates the height-11 integer matrix

$$B_{4,\text{free}}^\top = \begin{pmatrix} 3 & -6 & 0 & 1 \\ 8 & -9 & -2 & -10 \\ 11 & 8 & 3 & 2 \end{pmatrix} \in \mathbb{Z}^{3\times 4}$$

and lifting its three rows into $\ker(B_3^\top) \cap \ker(B_5^\top)$ determines a 3-dimensional subspace. We work with the following integer basis $B_4 \in \mathbb{Z}^{10\times 3}$ of that same subspace; it is related to the lifted rows by an invertible rational 3×3 change of basis, so P_4 in Eq. (7) is unchanged:

$$B_4^\top = \begin{pmatrix} 413 & -56 & 231 & 190 & -388 & -355 & 364 & 763 & -21 & -735 \\ 133 & 1052 & -185 & 407 & -189 & 98 & 917 & 1053 & 43 & -972 \\ 964 & -1150 & -393 & 1924 & 1946 & 1736 & -868 & -1858 & -246 & 426 \end{pmatrix}. \quad (10)$$

Geometric mechanism (principal-angle mode splitting). For two orthogonal projectors P, Q, Jordan's lemma implies $\|[P,Q]\|_\infty = \max_a c_a\sqrt{1-c_a^2}$, where $c_a \in [0,1]$ are the principal cosines (singular values of PQ). In the rank-1 qutrit KCBS optimum, the single principal cosine along every chord is $c = (\sqrt{5}-1)/2 \approx 0.6180$, sitting near the maximum $1/2$ of $f(c) = c\sqrt{1-c^2}$ at $c = 1/\sqrt{2} \approx 0.7071$, which yields $\mathcal{T}(P_i, P_{i+2}) \approx 0.4859$. In contrast, for our rank-3 projectors on $\mathbb{R}^{10}$, the three principal cosines along each chord $(i, i+2)$ split across the peak $c = 1/\sqrt{2}$ into one large overlap $c_{i,3} \approx 0.912$–0.918 and two smaller overlaps $c_{i,2} \approx 0.387$–0.404, $c_{i,1} \approx 0.116$–0.198. Because $f(c)$ decreases as $c \to 1$ as well as $c \to 0$, both $f(c_{i,3})$ and $f(c_{i,2})$ remain bounded below 0.374, suppressing $\max_i \mathcal{T}(P_i, P_{i+2})\,\mathcal{T}(P_{i+1}, P_{i+3})$ while the high-overlap channel $c_{i,3} \approx 0.915$ sustains constructive interference $\lambda_{\max}(\sum_{i=1}^5 P_i) > 2.1399$.

3 Exact Rational Certification

To rule out any floating-point or eigensolver inaccuracy, Theorem 1 is proved entirely in exact rational arithmetic over $\mathbb{Q}$ (reproducible via the accompanying standalone verification script).

Proof of Theorem 1. Step 1 (Projector premises and cyclic orthogonality). Direct integer arithmetic gives $\det(B_i^\top B_i) > 0$ for each $i \in \{1, \ldots, 5\}$; specifically,

$$\det(B_1^\top B_1) = 1, \quad \det(B_2^\top B_2) = 1, \quad \det(B_3^\top B_3) = 231007345, \quad \det(B_5^\top B_5) = 249409955,$$

and $\det(B_4^\top B_4) = 52113247544206186857 > 0$. Therefore each 3×3 Gram matrix $G_i = B_i^\top B_i$ is invertible over $\mathbb{Q}$, so $P_i = B_i G_i^{-1} B_i^\top \in M_{10}(\mathbb{Q})$ satisfies symmetry $P_i^\top = P_i$, idempotence $P_i^2 = B_i G_i^{-1} G_i G_i^{-1} B_i^\top = P_i$, and $\operatorname{rank}(P_i) = \operatorname{Tr}(P_i) = \operatorname{Tr}(I_3) = 3$. Moreover, exact integer matrix multiplication verifies $B_i^\top B_{i+1} = 0_{3\times 3}$ for all $i \in \mathbb{Z}_5$ (including $B_5^\top B_1 = 0_{3\times 3}$), whence

$$P_i P_{i+1} = B_i G_i^{-1} \big(B_i^\top B_{i+1}\big) G_{i+1}^{-1} B_{i+1}^\top = 0_{10\times 10} \qquad (\forall i \in \mathbb{Z}_5).$$

Step 2 (Exact Rayleigh lower bound on $\lambda_{\max}(\sum_{i=1}^5 P_i)$). Let $S := \sum_{i=1}^5 P_i \in M_{10}(\mathbb{Q})$ and choose the nonzero integer witness vector

$$v = (75, -13, 19, 76, -13, 12, 13, 20, -20, 36)^\top \in \mathbb{Z}^{10}, \qquad v^\top v = 14509 > 0. \tag{11}$$

By the variational characterization of the largest eigenvalue of a real symmetric matrix,

$$\lambda_{\max}(S) \geq L := \frac{v^\top S v}{v^\top v} = \frac{18644542173689482729982442027285568207177}{87127486824200215316298854938646553496 35},$$

which yields Eq. (4):

$$\begin{aligned} \lambda_{\max}(S) - 2 \geq L - 2 &= \frac{12190448088494396667226710395562575079 07}{87127486824200215316298854938646553496 35} \\ &\approx 0.1399150662. \end{aligned}$$

Step 3 (Exact PSD upper bounds on chord commutator norms). For each chord $i \in \{1, \ldots, 5\}$ (pairing P_i with P_{i+2} modulo 5), set $K_i := [P_i, P_{i+2}] = P_i P_{i+2} - P_{i+2} P_i \in M_{10}(\mathbb{Q})$. Because P_i and P_{i+2} are real symmetric, $K_i^\top = -K_i$, so $K_i^\top K_i = -K_i^2 \succeq 0$. Hence, for any rational $t_i \geq 0$,

$$\mathcal{T}(P_i, P_{i+2}) = \|K_i\|_\infty \leq t_i \quad \Longleftrightarrow \quad M_i := t_i^2 I_{10} + K_i^2 \succeq 0. \tag{12}$$

Define the five positive rational upper bounds

$$(t_1, t_2, t_3, t_4, t_5) = \left(\frac{187}{500}, \frac{183}{500}, \frac{149}{400}, \frac{183}{500}, \frac{729}{2000}\right) = (0.374,\, 0.366,\, 0.3725,\, 0.366,\, 0.3645). \tag{13}$$

For each $i \in \{1, \ldots, 5\}$, exact symmetric-pivoted Schur-complement ($LDL^\top$) reduction over $\mathbb{Q}$ confirms that $M_i = t_i^2 I_{10} + K_i^2$ has strictly positive rational pivots at all 10 elimination steps, proving $M_i \succeq 0$ (indeed $M_i \succ 0$) and hence $\mathcal{T}(P_i, P_{i+2}) \leq t_i$.

Step 4 (Crossing-chord comparison). The five adjacent products $t_i t_{i+1}$ ($i \in \mathbb{Z}_5$) are

$$\left(\frac{34221}{250000}, \frac{27267}{200000}, \frac{27267}{200000}, \frac{133407}{1000000}, \frac{136323}{1000000}\right),$$

whose maximum is $U_{\text{rhs}} := \max_{i\in\mathbb{Z}_5} t_i t_{i+1} = t_1 t_2 = \frac{34221}{250000} = 0.136884$, establishing Eq. (5). Subtracting U_{rhs} from $L - 2$ gives Eq. (6):

$$\lambda_{\max}(S) - 2 - \max_{i\in\mathbb{Z}_5} \mathcal{T}(P_i, P_{i+2})\, \mathcal{T}(P_{i+1}, P_{i+3}) \geq (L-2) - U_{\text{rhs}} = V > 0. \qquad \square$$

Remark 1 (Certified ratio lower bound versus true operator-norm ratio). As summarized in Table 1, one should distinguish two slightly different ratios associated with this counterexample. Dividing the rational Rayleigh lower bound $L - 2$ by the certified rational upper bound $U_{\text{rhs}} = \max_{i\in\mathbb{Z}_5} t_i t_{i+1}$ gives the rigorous rational lower bound $(L-2)/U_{\text{rhs}} \approx 1.02214$ (≈ 1.0221). Evaluating the exact eigenvalues of S and $-K_i^2$ numerically gives $\lambda_{\max}(S) - 2 \approx 0.139957$ and $\max_{i\in\mathbb{Z}_5} \mathcal{T}(P_i, P_{i+2})\, \mathcal{T}(P_{i+1}, P_{i+3}) \approx 0.136806$, corresponding to a true operator-norm ratio of ≈ 1.02303 (≈ 1.023) and a true additive gap of ≈ 0.00315.

Table 1: Comparison between the exact rational certificate bounds used in the proof of Theorem 1 and the numerical true operator-norm values for the same rank-3, $d = 10$ projectors $P_1, \ldots, P_5$ (here $S := \sum_{i=1}^{5} P_i$, $\mathcal{T}_i := \mathcal{T}(P_i, P_{i+2})$, LHS $:= \lambda_{\max}(S) - 2$, RHS $:= \max_{i \in \mathbb{Z}_5} \mathcal{T}_i \mathcal{T}_{i+1}$, and LB/UB denote certified lower/upper bounds).

Quantity	Exact rational certificate	Numerical spectral value
Pentagon excess	$L - 2 \approx 0.13991507$ (LB)	$\lambda_{\max}(S) - 2 \approx 0.13995741$
Chord scores (t_i vs. $\mathcal{T}_i$)	$(0.374, 0.366, 0.3725, 0.366, 0.3645)$	$(0.37394, 0.36585, 0.37233, 0.36552, 0.36442)$
Crossing-chord RHS	$U_{\text{rhs}} = \frac{34221}{250000} = 0.136884$ (UB)	$\max_i \mathcal{T}_i \mathcal{T}_{i+1} \approx 0.13680614$
Additive violation	$V = (L - 2) - U_{\text{rhs}} \approx +0.00303107$	LHS $-$ RHS $\approx +0.00315127$
LHS-to-RHS ratio	$(L - 2)/U_{\text{rhs}} \approx 1.02214332$ (LB)	LHS/RHS ≈ 1.02303456

Standalone reproducibility and formal verification. All verification artifacts are publicly archived online:[1]

- **Exact rational Python script** (`verify_counterexample.py`): zero external dependencies (using only `fractions.Fraction` from the Python 3 standard library). Starting solely from $B_1, \ldots, B_5$ in Eqs. (8)–(10), v in Eq. (11), and t in Eq. (13), it reconstructs $P_1, \ldots, P_5$ via Eq. (7) and verifies all projector premises, cyclic orthogonality, pivoted $LDL^\top$ positive-semidefiniteness of $M_1, \ldots, M_5$, and the exact fraction V in Eq. (6) in under 0.1 seconds.
- **Lean 4 / Mathlib formalization** (`VerifyCounterexample.lean`, Lean 4.36.0): formalizes Wu's Conjecture 2 directly in its commutator operator-norm form (`WuPentagonBound`, obtained via Wu's Theorem 1 identity (2) rather than formalizing Lüders-instrument total-variation optimization from scratch). Its top-level theorems `wu_conj2_counterexample_dim10_rank3` and `disproof_wu_conj2` ($\neg$ `WuPentagonBound`) kernel-verify the existence of five rank-3 orthogonal projectors in Matrix(Fin 10, Fin 10, $\mathbb{C}$) achieving the exact margin V, backed by the internal real-rational construction `WuKcbsCounterexample.P` and separate bound lemmas `P_lhs_lower_bound` and `P_rhs_upper_bound`, with zero `sorry` and standard Lean axioms only (`propext`, `Classical.choice`, `Quot.sound`).

4 Scope, Limitations, and Open Questions

We emphasize the exact scope of Theorem 1 so as not to overclaim beyond what the certificate establishes:

1. **Unaffected proved results:** Theorem 10 of Wu [4], which proves the pentagon bound (3) for rank-1 projectors on $\mathbb{C}^3$ and arbitrary direct sums thereof, is completely unaffected. Likewise, other results and conjectures in Ref. [4] (such as Theorem 1 and the qubit Jensen–Shannon monotonicity problem in §II.C) are distinct from Wu's Conjecture 2.
2. **Rank-1 regime remains open:** Theorem 1 relies crucially on rank-3 principal-angle splitting ($r_i = 3$). Whether the pentagon bound (3) holds for *rank-1 projectors* in all dimensions $d \geq 4$ (equivalently $d \in \{4, 5\}$, since five rank-1 ranges span a subspace of dimension at most 5) remains open.
3. **Dimensions $4 \leq d \leq 9$ remain open:** Our counterexample acts on $\mathbb{R}^{10} \subset \mathbb{C}^{10}$ with uniform ranks $(3, 3, 3, 3, 3)$ (and extends to $d > 10$ by zero-padding). Whether any counterexample exists in dimensions $4 \leq d \leq 9$—including rank-2 projectors in $d \in \{6, 7, 8\}$ or rank-3 projectors in $d = 9$—remains unresolved.

[1] `https://github.com/kadeee0711/research-notes/tree/main/quantum-information/2026-11-wu-kcbs-pentagon-counterexample`

4. **Sharp constant for arbitrary rank:** Whether the pentagon bound (3) holds up to some multiplicative constant C, i.e. $\lambda_{\max}(\sum_{i=1}^{5} P_i) - 2 \leq C \max_{i\in\mathbb{Z}_5} \mathcal{T}(P_i, P_{i+2})\,\mathcal{T}(P_{i+1}, P_{i+3})$ for all orthogonal projectors with $P_iP_{i+1} = 0$ (with C universal or depending on (d, r)), and if so what the optimal constant is, remains open; Theorem 1 shows that any such constant must satisfy $C > 1.02214$ (numerically $C \gtrsim 1.02303$).

References

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